Pre-AP Geometry 11.2

Notes Areas of Regular Polygons (pp 669-71)

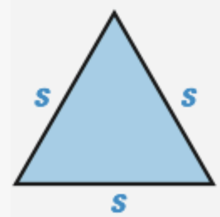
- **I can find the area of an equilateral triangle.**
- **I can find the area of the regular polygons.**

THEOREM

THEOREM 11.3 ***Area of an Equilateral Triangle***

The area of an equilateral triangle is one fourth the square of the length of the side times $\sqrt{3}$.

$$A = \frac{1}{4}\sqrt{3}s^2$$



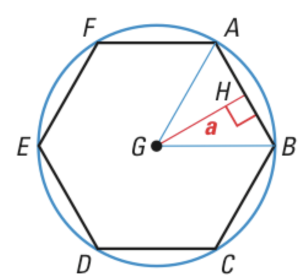
THEOREM

THEOREM 11.4 ***Area of a Regular Polygon***

The area of a regular n-gon with side length s is half the product of the apothem a and the perimeter P, so $A = \frac{1}{2}aP$, or $A = \frac{1}{2}a \cdot ns$.

The **center of the polygon** and **radius of the polygon** are the center and radius of its circumscribed circle, respectively.

The distance from the center to any side of the polygon is called the **apothem of the polygon**. The apothem is the height of a triangle between the center and two consecutiv vertices of the polygon.



Hexagon *ABCDEF* with center *G*, radius *GA*, and apothem *GH*

$$A = \boxed{\text{area of one triangle}} \cdot \boxed{\text{number of triangles}}$$

$$= \left(\frac{1}{2} \cdot \text{apothem} \cdot \text{side length } s\right) \cdot \text{number of sides}$$

$$= \frac{1}{2} \cdot \text{apothem} \cdot \text{number of sides} \cdot \text{side length } s$$

$$= \frac{1}{2} \cdot \text{apothem} \cdot \text{perimeter of polygon}$$

The number of congruent triangles formed will be the same as the number of sides of the polygon.

STUDENT HELP

Study Tip
In a regular polygon, the length of each side is the same. If this length is s and there are n sides, then the perimeter P of the polygon is $n \cdot s$, or $P = ns$.

This approach can be used to find the area of any regular polygon.

Notes Areas of Regular Polygons (pp 669-71)

A **central angle of a regular polygon** is an angle whose vertex is the center and whose sides contain two consecutive vertices of the polygon. You can divide 360° by the number of sides to find the measure of each central angle of the polygon.

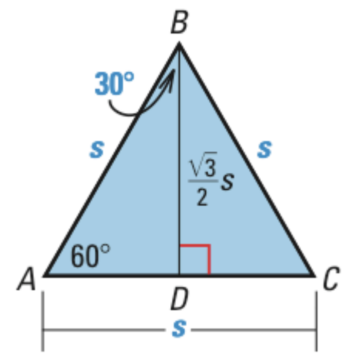
EXAMPLE 1 *Proof of Theorem 11.3*

Prove Theorem 11.3. Refer to the figure below.

SOLUTION

GIVEN ▶ $\triangle ABC$ is equilateral.

PROVE ▶ Area of $\triangle ABC$ is $A = \frac{1}{4}\sqrt{3}\,s^2$.



Paragraph Proof Draw the altitude from B to side $\overline{AC}$. Then $\triangle ABD$ is a 30°-60°-90° triangle. From Lesson 9.4, the length of $\overline{BD}$, the side opposite the 60° angle in $\triangle ABD$, is $\frac{\sqrt{3}}{2}s$. Using the formula for the area of a triangle,

$$A = \frac{1}{2}bh = \frac{1}{2}(s)\left(\frac{\sqrt{3}}{2}s\right) = \frac{1}{4}\sqrt{3}\,s^2.$$

EXAMPLE 2 *Finding the Area of an Equilateral Triangle*

Find the area of an equilateral triangle with 8 inch sides.

SOLUTION

Use $s = 8$ in the formula from Theorem 11.3.

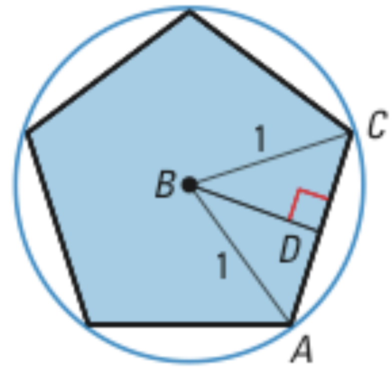
$$A = \frac{1}{4}\sqrt{3}s^2 = \frac{1}{4}\sqrt{3}\,(8^2) = \frac{1}{4}\sqrt{3}\,(64) = \frac{1}{4}(64)\sqrt{3} = 16\sqrt{3} \text{ square inches}$$

▶ Using a calculator, the area is about 27.7 square inches.

6. Find the area of equilateral triangle whose perimeter is 6 in.

EXAMPLE 3 Finding the Area of a Regular Polygon

A regular pentagon is inscribed in a circle with radius 1 unit. Find the area of the pentagon.



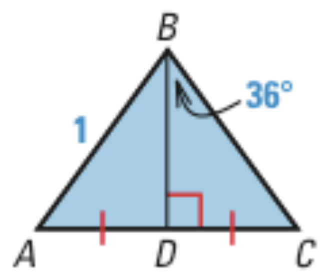
SOLUTION

To apply the formula for the area of a regular pentagon, you must find its apothem and perimeter.

The measure of central $\angle ABC$ is $\frac{1}{5} \cdot 360°$, or $72°$.

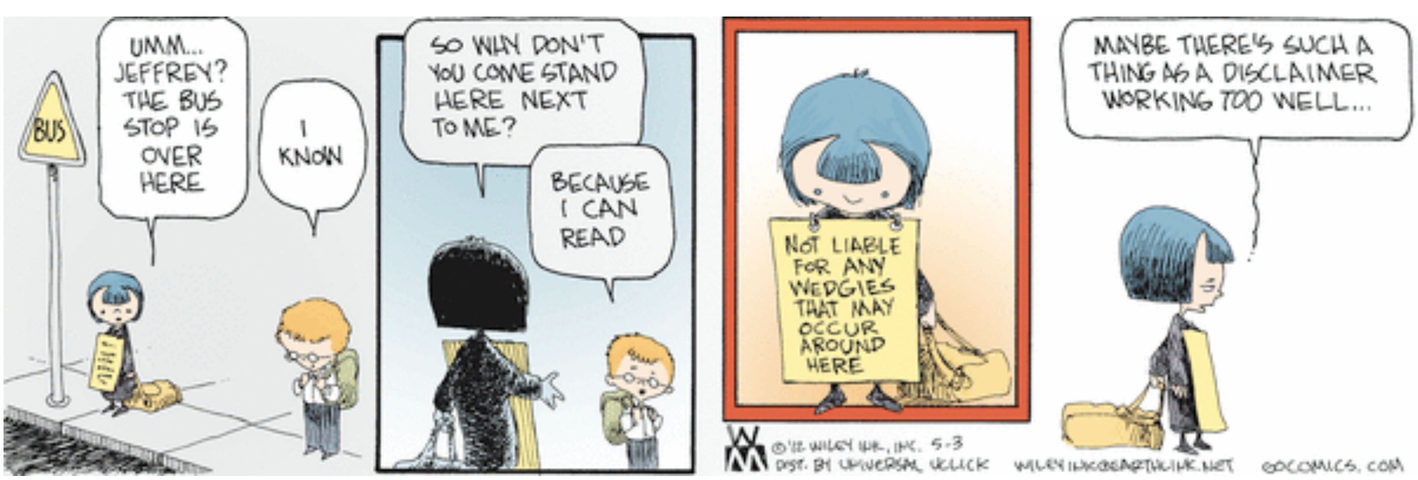
In isosceles triangle $\triangle ABC$, the altitude to base $\overline{AC}$ also bisects $\angle ABC$ and side $\overline{AC}$. The measure of $\angle DBC$, then, is $36°$. In right triangle $\triangle BDC$, you can use trigonometric ratios to find the lengths of the legs.

$$\cos 36° = \frac{BD}{BC} = \frac{BD}{1} = BD \qquad \sin 36° = \frac{DC}{BC} = \frac{DC}{1} = DC$$



▶ So, the pentagon has an apothem of $a = BD = \cos 36°$ and a perimeter of $P = 5(AC) = 5(2 \cdot DC) = 10 \sin 36°$. The area of the pentagon is

$$A = \frac{1}{2}aP = \frac{1}{2}(\cos 36°)(10 \sin 36°) \approx 2.38 \text{ square units.}$$



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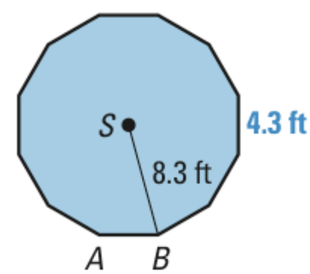
EXAMPLE 4 *Finding the Area of a Regular Dodecagon*

PENDULUMS The enclosure on the floor underneath the Foucault Pendulum at the Houston Museum of Natural Sciences in Houston, Texas, is a regular dodecagon with a side length of about 4.3 feet and a radius of about 8.3 feet. What is the floor area of the enclosure?



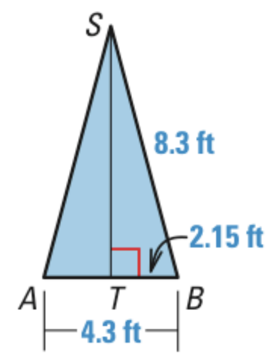
SOLUTION

A dodecagon has 12 sides. So, the perimeter of the enclosure is

$$P \approx 12(4.3) = 51.6 \text{ feet.}$$

In $\triangle SBT$, $BT = \frac{1}{2}(BA) = \frac{1}{2}(4.3) = 2.15$ feet. Use the Pythagorean Theorem to find the apothem ST.

$$a = \sqrt{8.3^2 - 2.15^2} \approx 8 \text{ feet}$$



▶ So, the floor area of the enclosure is

$$A = \frac{1}{2}aP \approx \frac{1}{2}(8)(51.6) = 206.4 \text{ square feet.}$$

9. What is the area of a regular 9-gon with a side length of 6 in.?

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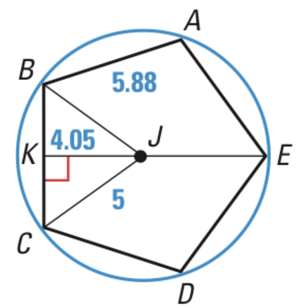
6. Identify the center of polygon ABCDE.



7. Identify the radius of the polygon.

8. Identify a central angle of the polygon.

9. Identify a segment whose length is the apothem.

10. In a regular polygon, how do you find the measure of the central angle?

11. What is the area of an equilateral triangle with 3 inch sides?

The stop sign shown is a regular octagon. Its perimeter is about 80 inches and its height is about 24 inches.

12. What is the measure of each central angle?

13. Find the apothem, radius, and area of the stop sign.